9.2 The Pythagorean Theorem

GOAL 1 Proving the Pythagorean Theorem

What you should learn

GOAL 1 Prove the Pythagorean Theorem.

GOAL 2 Use the Pythagorean Theorem to solve **real-life** problems, such as determining how far a ladder will reach in **Ex. 32**.

Why you should learn it

▼ To measure **real-life** lengths indirectly, such as the length of the support beam of a skywalk in **Example 4**.

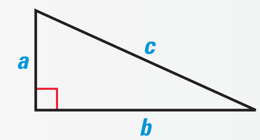
REAL LIFE

In this lesson, you will study one of the most famous theorems in mathematics—the *Pythagorean Theorem*. The relationship it describes has been known for thousands of years.

THEOREM

THEOREM 9.4 ***Pythagorean Theorem***

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs.



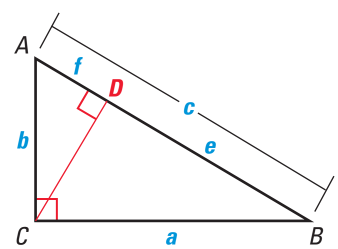
$c^2 = a^2 + b^2$

PROVING THE PYTHAGOREAN THEOREM There are many different proofs of the Pythagorean Theorem. One is shown below. Other proofs are found in Exercises 37 and 38 on page 540, and in the *Math and History* feature on page 557.

GIVEN ▶ In $\triangle ABC$, $\angle BCA$ is a right angle.

PROVE ▶ $a^2 + b^2 = c^2$

Plan for Proof Draw altitude $\overline{CD}$ to the hypotenuse. Then apply Geometric Mean Theorem 9.3, which states that when the altitude is drawn to the hypotenuse of a right triangle, each leg of the right triangle is the geometric mean of the hypotenuse and the segment of the hypotenuse that is adjacent to that leg.



Proof

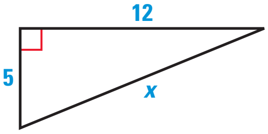
Statements	Reasons
1. Draw a perpendicular from C to $\overline{AB}$.	**1.** Perpendicular Postulate
2. $\frac{c}{a} = \frac{a}{e}$ and $\frac{c}{b} = \frac{b}{f}$	**2.** Geometric Mean Theorem 9.3
3. $ce = a^2$ and $cf = b^2$	**3.** Cross product property
4. $ce + cf = a^2 + b^2$	**4.** Addition property of equality
5. $c(e + f) = a^2 + b^2$	**5.** Distributive property
6. $e + f = c$	**6.** Segment Addition Postulate
7. $c^2 = a^2 + b^2$	**7.** Substitution property of equality

GOAL 2 USING THE PYTHAGOREAN THEOREM

A **Pythagorean triple** is a set of three positive integers a, b, and c that satisfy the equation $c^2 = a^2 + b^2$. For example, the integers 3, 4, and 5 form a Pythagorean triple because $5^2 = 3^2 + 4^2$.

EXAMPLE 1 *Finding the Length of a Hypotenuse*

Find the length of the hypotenuse of the right triangle. Tell whether the side lengths form a Pythagorean triple.



SOLUTION

$$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$$ **Pythagorean Theorem**

$$x^2 = 5^2 + 12^2$$ **Substitute.**

$$x^2 = 25 + 144$$ **Multiply.**

$$x^2 = 169$$ **Add.**

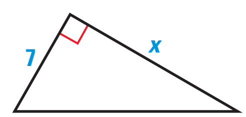
$$x = 13$$ **Find the positive square root.**

▶ Because the side lengths 5, 12, and 13 are integers, they form a Pythagorean triple.

Many right triangles have side lengths that do not form a Pythagorean triple, as shown in Example 2.

EXAMPLE 2 *Finding the Length of a Leg*

Find the length of the leg of the right triangle.



SOLUTION

$$(\text{hypotenuse})^2 = (\text{leg})^2 + (\text{leg})^2$$ **Pythagorean Theorem**

$$14^2 = 7^2 + x^2$$ **Substitute.**

$$196 = 49 + x^2$$ **Multiply.**

$$147 = x^2$$ **Subtract 49 from each side.**

$$\sqrt{147} = x$$ **Find the positive square root.**

$$\sqrt{49} \cdot \sqrt{3} = x$$ **Use product property.**

$$7\sqrt{3} = x$$ **Simplify the radical.**

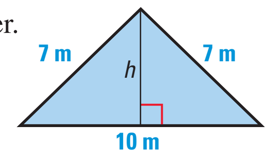
STUDENT HELP

Skills Review
For help with simplifying radicals, see p. 799.

In Example 2, the side length was written as a radical in simplest form. In real-life problems, it is often more convenient to use a calculator to write a decimal approximation of the side length. For instance, in Example 2, $x = 7 \cdot \sqrt{3} \approx 12.1$.

EXAMPLE 3 *Finding the Area of a Triangle*

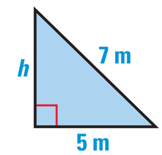
Find the area of the triangle to the nearest tenth of a meter.



SOLUTION

You are given that the base of the triangle is 10 meters, but you do not know the height h.

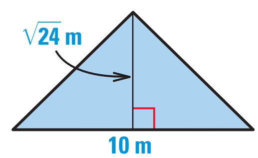
Because the triangle is isosceles, it can be divided into two congruent right triangles with the given dimensions. Use the Pythagorean Theorem to find the value of h.



$7^2 = 5^2 + h^2$	**Pythagorean Theorem**
$49 = 25 + h^2$	**Multiply.**
$24 = h^2$	**Subtract 25 from both sides.**
$\sqrt{24} = h$	**Find the positive square root.**

Now find the area of the original triangle.

$$\text{Area} = \frac{1}{2}bh$$
$$= \frac{1}{2}(10)(\sqrt{24})$$
$$\approx 24.5 \text{ m}^2$$



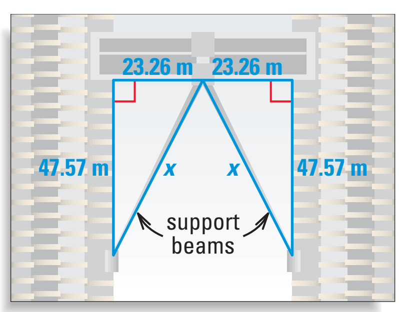
▶ The area of the triangle is about 24.5 m^2.

STUDENT HELP

Look Back
For help with finding the area of a triangle, see p. 51.

EXAMPLE 4 *Indirect Measurement*

SUPPORT BEAM The skyscrapers shown on page 535 are connected by a skywalk with support beams. You can use the Pythagorean Theorem to find the approximate length of each support beam.

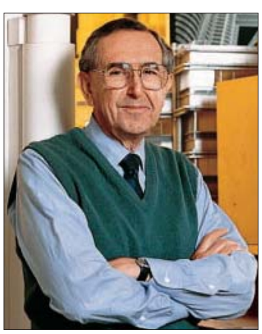


Each support beam forms the hypotenuse of a right triangle. The right triangles are congruent, so the support beams are the same length.

$x^2 = (23.26)^2 + (47.57)^2$	**Pythagorean Theorem**
$x = \sqrt{(23.26)^2 + (47.57)^2}$	**Find the positive square root.**
$x \approx 52.95$	**Use a calculator to approximate.**

▶ The length of each support beam is about 52.95 meters.

FOCUS ON PEOPLE



REAL LIFE

CESAR PELLI is an architect who designed the twin skyscrapers shown on page 535. These 1483 foot buildings tower over the city of Kuala Lumpur, Malaysia.

GUIDED PRACTICE

Vocabulary Check ✓

1. State the Pythagorean Theorem in your own words.

Concept Check ✓

2. Which equations are true for $\triangle PQR$?

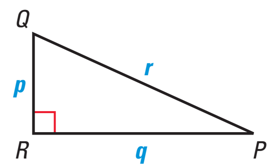
A. $r^2 = p^2 + q^2$

B. $q^2 = p^2 + r^2$

C. $p^2 = r^2 - q^2$

D. $r^2 = (p + q)^2$

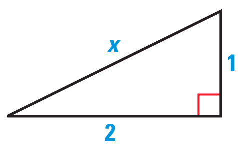
E. $p^2 = q^2 + r^2$



Skill Check ✓

Find the unknown side length. Tell whether the side lengths form a Pythagorean triple.

3.



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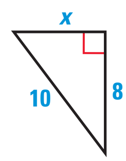

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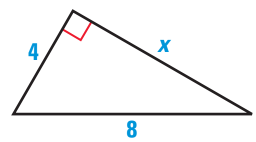
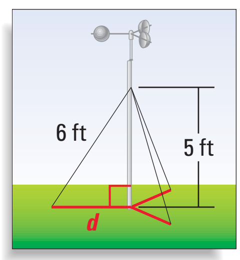


6. **ANEMOMETER** An anemometer (an uh MAHM ih tur) is a device used to measure windspeed. The anemometer shown is attached to the top of a pole. Support wires are attached to the pole 5 feet above the ground. Each support wire is 6 feet long. How far from the base of the pole is each wire attached to the ground?



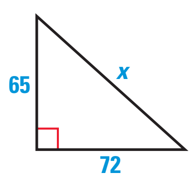
PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 819.

FINDING SIDE LENGTHS **Find the unknown side length. Simplify answers that are radicals. Tell whether the side lengths form a Pythagorean triple.**

7.



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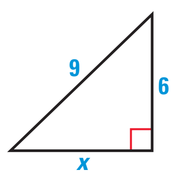

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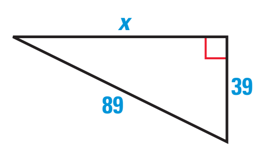

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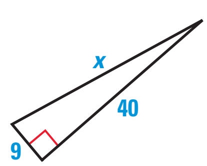

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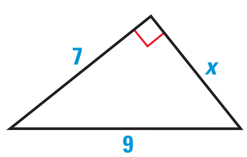

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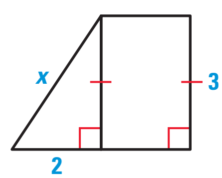

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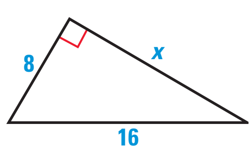

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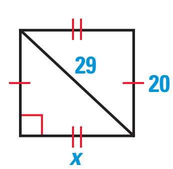

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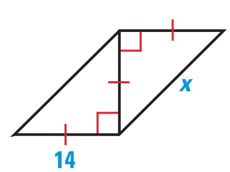

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 7–24
Example 2: Exs. 7–24
Example 3: Exs. 25–30
Example 4: Exs. 31–36

FINDING LENGTHS **Find the value of *x*. Simplify answers that are radicals.**

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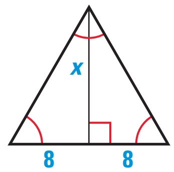

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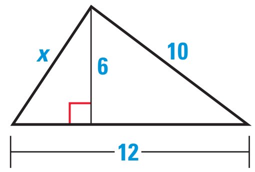

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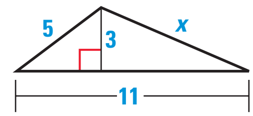

PYTHAGOREAN TRIPLES **The variables *r* and *s* represent the lengths of the legs of a right triangle, and *t* represents the length of the hypotenuse. The values of *r, s,* and *t* form a Pythagorean triple. Find the unknown value.**

19. $r = 12, s = 16$ **20.** $r = 9, s = 12$ **21.** $r = 18, t = 30$

22. $s = 20, t = 101$ **23.** $r = 35, t = 37$ **24.** $t = 757, s = 595$

STUDENT HELP

Look Back
For help with finding areas of quadrilaterals, see pp. 372–375.

FINDING AREA **Find the area of the figure. Round decimal answers to the nearest tenth.**

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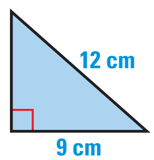

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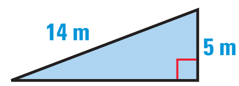

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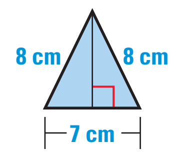

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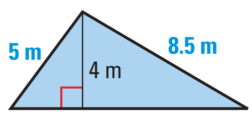

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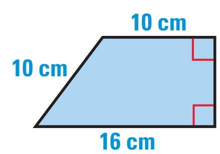

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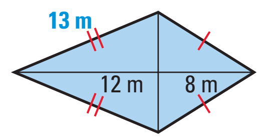
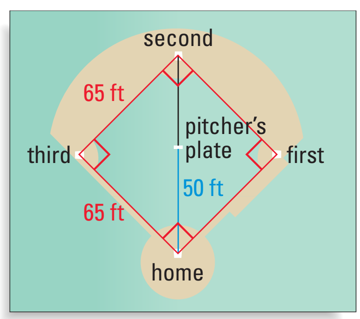
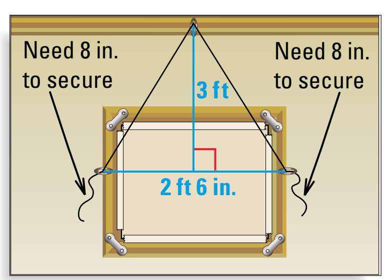


31. **SOFTBALL DIAMOND** In slow-pitch softball, the distance between consecutive bases is 65 feet. The pitcher's plate is located on a line between second base and home plate, 50 feet from home plate. How far is the pitcher's plate from second base? Justify your answer.



32. **SAFETY** The distance of the base of a ladder from the wall it leans against should be at least $\frac{1}{4}$ of the ladder's total length. Suppose a 10 foot ladder is placed according to these guidelines. Give the minimum distance of the base of the ladder from the wall. How far up the wall will the ladder reach? Explain. Include a sketch with your explanation.

33. **ART GALLERY** You want to hang a painting 3 feet from a hook near the ceiling of an art gallery, as shown. In addition to the length of wire needed for hanging, you need 16 inches of wire to secure the wire to the back of the painting. Find the total length of wire needed to hang the painting.



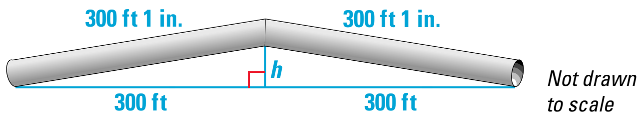
FOCUS ON CAREERS

REAL LIFE

MECHANICAL ENGINEERS use science, mathematics, and engineering principles in their work. They evaluate, install, operate, and maintain mechanical products and systems, such as the Trans-Alaska pipeline.

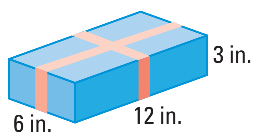
INTERNET **CAREER LINK** www.mcdougallittell.com

34. **TRANS-ALASKA PIPELINE** Metal expands and contracts with changes in temperature. The Trans-Alaska pipeline was built to accommodate expansion and contraction. Suppose that it had not been built this way. Consider a 600 foot section of pipe that expands 2 inches and buckles, as shown below. Estimate the height h of the buckle.

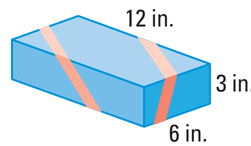


WRAPPING A BOX **In Exercises 35 and 36, two methods are used to wrap ribbon around a rectangular box with the dimensions shown below. The amount of ribbon needed does not include a knot or bow.**

Method 1

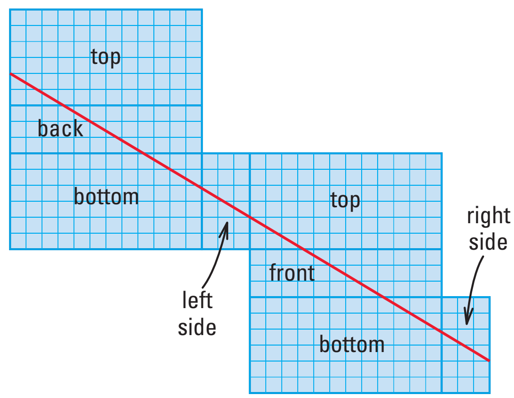


Method 2



35. How much ribbon is needed to wrap the box using Method 1?

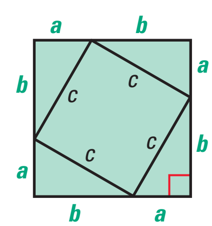
36. The red line on the diagram at the right shows the path the ribbon follows around the box when Method 2 is used. Does Method 2 use more or less ribbon than Method 1? Explain your thinking.



STUDENT HELP

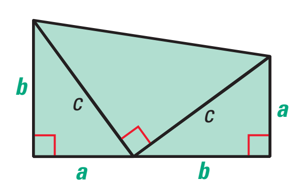
INTERNET **HOMEWORK HELP** Visit our Web site www.mcdougallittell.com for help with the proofs in Exs. 37 and 38.

37. **PROVING THE PYTHAGOREAN THEOREM** Explain how the diagram at the right can be used to prove the Pythagorean Theorem algebraically. (*Hint:* Write two different expressions that represent the area of the large square. Then set them equal to each other.)



38. **GARFIELD'S PROOF** James Abram Garfield, the twentieth president of the United States, discovered a proof of the Pythagorean Theorem in 1876. His proof involved the fact that a trapezoid can be formed from two congruent right triangles and an isosceles right triangle.

Use the diagram to write a paragraph proof showing that $a^2 + b^2 = c^2$. (*Hint:* Write two different expressions that represent the area of the trapezoid. Then set them equal to each other.)



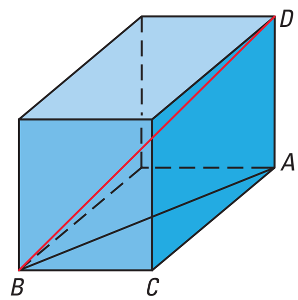
Test Preparation

39. MULTI-STEP PROBLEM To find the length of a diagonal of a rectangular box, you can use the Pythagorean Theorem twice. Use the theorem once with right $\triangle ABC$ to find the length of the diagonal of the base.

$$AB = \sqrt{(AC)^2 + (BC)^2}$$

Then use the theorem with right $\triangle ABD$ to find the length of the diagonal of the box.

$$BD = \sqrt{(AB)^2 + (AD)^2}$$



a. Is it possible to carry a 9 foot piece of lumber in an enclosed rectangular trailer that is 4 feet by 8 feet by 4 feet?

b. Is it possible to store a 20 foot long pipe in a rectangular room that is 10 feet by 12 feet by 8 feet? Explain.

c. *Writing* Write a formula for finding the diagonal d of a rectangular box with length ℓ, width w, and height h. Explain your reasoning.

★ Challenge

PERIMETER OF A RHOMBUS The diagonals of a rhombus have lengths *a* and *b*. Use this information in Exercises 40 and 41.

40. Prove that the perimeter of the rhombus is $2\sqrt{a^2 + b^2}$.

41. The perimeter of a rhombus is 80 centimeters. The lengths of its diagonals are in the ratio 3 : 4. Find the length of each diagonal.

EXTRA CHALLENGE
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MIXED REVIEW

USING RADICALS Evaluate the expression. (Algebra Review, p. 522, for 9.3)

42. $(\sqrt{6})^2$ **43.** $(\sqrt{9})^2$ **44.** $(\sqrt{14})^2$ **45.** $(2\sqrt{2})^2$

46. $(4\sqrt{13})^2$ **47.** $-(5\sqrt{49})^2$ **48.** $4(\sqrt{9})^2$ **49.** $(-7\sqrt{3})^2$

LOGICAL REASONING Determine whether the true statement can be combined with its converse to form a true biconditional statement. (Review 2.2)

50. If a quadrilateral is a square, then it has four congruent sides.

51. If a quadrilateral is a kite, then it has two pairs of congruent sides.

52. For all real numbers x, if $x \geq 1$, then $x^2 \geq 1$.

53. For all real numbers x, if $x > 1$, then $\frac{1}{x} < 1$.

54. If one interior angle of a triangle is obtuse, then the sum of the other two interior angles is less than 90°.

USING ALGEBRA Prove that the points represent the vertices of a parallelogram. (Review 6.3)

55. $P(4, 3)$, $Q(6, -8)$, $R(10, -3)$, $S(8, 8)$

56. $P(5, 0)$, $Q(2, 9)$, $R(-6, 6)$, $S(-3, -3)$